Time preserving tomography

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Summary

Time Preserving Tomography is an accurate, efficient and flexible tool for constructing kinematically equivalent subsurface models given a background model and a set of model parameter perturbations. In this method, parameters of the background model are allowed to change while preserving traveltimes of all ray pairs. Perturbations can be applied for all types of model parameters. In the case of tilted transverse isotropy (TTI) the model parameters are the axial compressional velocity, Thomsen anisotropic interval parameters epsilon and delta, and the depth values of the model horizons. The traveltimes of all ray pairs traced during the tomographic inversion are kinematic invariants. In the migrated domain, all the kinematically equivalent models should provide more or less flat reflection events along common image gathers (CIG). In fact, time-preserving tomography does not require CIG at all. An example of the application of this type of method is the use of misties between well markers and seismic depth horizons to obtain Thomsen delta parameters (Mancini, 2013). Time-preserving tomography is a very useful tool for depth interpretation, uncertainty analysis and risk management.

Introduction

Subsurface geological models derived from seismic imaging are not unique. Many combinations of subsurface geological model representations and model parameters can be used to satisfy the imaging conditions and flatten common image gathers. This is mainly due to the limitations of seismic data acquisitions, in particular the limited offset between sources and receivers and the lack of azimuth coverage. Other factors for the non-uniqueness are finite frequency band, noise, poorly illuminated zones and limitations in the asymptotic high-frequency ray method and theory of wave propagation used (e.g. not accounting for factors like the "real" anisotropy, dispersion from attenuation, scattering, etc.). The process of deriving a subsurface anisotropic velocity model from seismic data is very demanding, normally requiring massive computational power and intensive human involvement. This non-uniqueness problem demonstrates the motivation and need for the following processes:

1. Integration of additional external information into the model building procedures in order to reduce the ambiguity / non-uniqueness. The most prominent example of this is well markers that are used to constrain the depths of the seismic horizons.
2. Analysis of equivalent subsurface models that differ from the current model by small perturbations in the model parameters. Starting with a given background model, we want to define small changes in one or more of the model parameters, and construct a new model that respects these small changes, while keeping the rays traveltimes unchanged (i.e. fully in agreement with the recorded seismic data). Such a construction is natural in the interpretation process (asking interpretative queries like, "What are the location changes of the interface horizons due to given perturbations of the axial compression velocity and Thomsen parameters?").

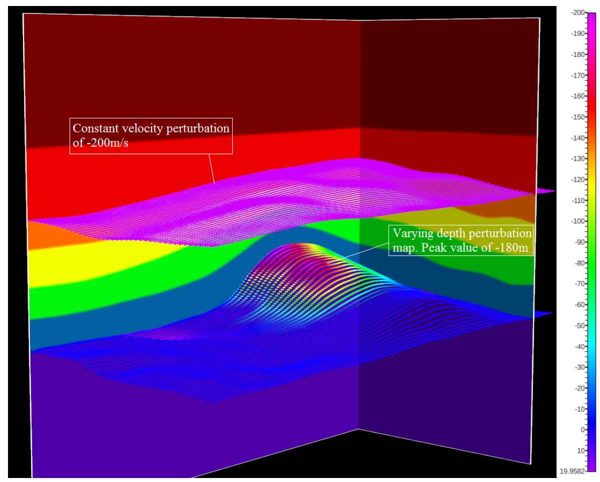


Figure 1: Background subsurface velocity model and model perturbation maps. The layer above the deepest horizon was assigned a laterally variant depth perturbation map (mistie map). Second formation from the top was assigned a constant velocity perturbation of $-200\mathrm{m/s}$.

We present the concept of Time Preserving Tomography - an accurate and efficient tool for constructing a kinematically equivalent subsurface model given an initial background model and a set of model perturbations (which we call constraints), while keeping the total traveltime along specular incident and reflected rays traveling through the subsurface model unchanged. Under the assumptions of tilted transverse isotropy (TTI), by model perturbations we mean small changes in the axial compressional velocity, anisotropic interval parameters epsilon and delta, and the depth values of the model horizons. In Time Preserving Tomography we first calculate a set of kinematic characteristics inherent to the background model. These kinematic characteristics are computed on a relatively coarse grid (the tomographic inversion grid) by calculating the discrete traveltime integrals along specular rays shot in wide opening angles and all azimuths through our background anisotropic model. This results in a set of linear homogeneous equations (the right hand side of the system describes traveltime errors which are set to zero in this case). The kinematic characteristics are the coefficients of the above-mentioned homogeneous system. These coefficients are stored in a matrix called the influence matrix. In the second stage we add the imposed model constraints (perturbations) to the influence matrix and solve the system using the Conjugate Gradient algorithm.

Key features of our system include:

Accuracy - the perturbed models preserve the total traveltime of all rays travelling through the model. In particular, normal incident traveltimes are preserved, while vertical time is generally not.

Flexibility - the system can receive perturbations for different model parameters for each subsurface geological layer. The perturbations are defined as layer parameter maps which can vary laterally for each subsurface layer.

User control - the user has full control over the inversion process. The constraints are added as soft constraints; that is, we don't set any variables to a specific value, but rather introduce the constraints as additional equations in our system, and find the best solution to the entire system in a least squares sense. The user can control the weight he/she wishes to assign to the constraints relative to the influence matrix. The user can define the reflection angle range for which traveltimes will be preserved. Accurate depth-to-depth conversion can be achieved by using only small angles around the Normal Incidence rays. A wider range (up to 30 degrees half-opening angles) may be used to convert isotropic models into anisotropic models using additional information about well-based depth misties.

Efficiency - the most time-consuming step is the first one, that is run only once for a given model. The specific constraints are defined and used only in the second part, which is fast (order of minutes for medium-size surveys). Therefore, the user can run several scenarios quickly to generate a series of equivalent models.

Theory

Ray-based residual traveltime tomography in general and time-preserving tomography in particular, is based on solving a very large, over-determined system of linear equations. These linear equations can be viewed as a set of linear constraints. In time-preserving tomography we distinguish between two types of linear constraints: (a) imposing zero traveltime errors along the traced rays and (b) explicitly setting model parameter perturbations for either (or both) horizons' depths and anisotropic velocity parameters. Solving this system of linear equations will result in an updated model that simultaneously satisfies both types of constraints.

Preserving Traveltimes

In ray-based reflection tomography (Kosloff et al., 1996; Woodward et al., 1998; Billette and Lambaré, 1998) an influence matrix is constructed, relating variations in medium parameters to residual traveltimes computed from residual moveouts (RMO) measured along CIG. In the case of time-preserving tomography, the assumption is that the starting point is a model which is consistent with the short offset data at least, and possibly with longer offset events. The data consistency can be viewed and defined with respect to the range of flattened events within the CIGs.

We use a model-based approach in which the subsurface model is defined by a set of geological layers separated by curved interfaces and faults. Our 3D grids for the model update parameters and for the shooting points for fans of ray pairs are regular in the horizontal directions, and irregular in the vertical direction - the grid points coincide with the structural interfaces. The updated model parameters are represented on a coarse grid. Rays are shot from a finer horizontal grid in which pairs of incident and reflected rays are shot with respect to the Normal direction. The traveltime variation with respect to the model parameters of each ray pair is given by

$$0 = \delta t = \sum_i (A_V^i \Delta V^i + A_\varepsilon^i \Delta\varepsilon^i + A_\delta^i \Delta\delta^i + A_z^i \Delta z^i) \qquad (1)$$

with i an index running on all model nodes. $\Delta V, \Delta\varepsilon, \Delta\delta, \Delta z$ are the variations (updates) of the axial compressional velocity, Thomsen parameters and the horizon depth values respectively. $A_V^i, A_\varepsilon^i, A_\delta^i, A_z^i$ are the components of the influence matrix and are given by

$$A_m^i = \frac{\partial t}{\partial m^i} \qquad (2)$$

with t the total ray pair traveltime and $m = V, \varepsilon, \delta, z$. Each ray pair produces such an equation; together they define an over-determined system of linear equations (the number of rays is considerably larger than the number of model nodes). This could be written in a matrix-vector notation,

$$A_V \Delta V + A_\varepsilon \Delta\varepsilon + A_\delta \Delta\delta + A_z \Delta z = 0 \qquad (3)$$

This equation represents our first type of traveltime preservation constraints. It can be brought to an even simpler form if we define

$$\Delta m = \begin{pmatrix} \Delta V \\ \Delta\varepsilon \\ \Delta\delta \\ \Delta z \end{pmatrix}, A = \begin{pmatrix} A_V & A_\varepsilon & A_\delta & A_z \end{pmatrix}; \quad A\Delta m = \delta t \qquad (4)$$

If the total number of ray pairs shot was N_R and the number of model parameters is N_m then A is a $N_R \times N_m$ matrix.

Introduction of Model Perturbations

In order to obtain a desired value $\tilde{m}^i$ of a model parameter at node i, we define a perturbation to the background value m^i

$$C_m^i = \tilde{m}^i - m^i \qquad (5)$$

Our second type of constraint is simply the demand that

$$\Delta m^i = C_m^i \qquad (6)$$

We may introduce such an equation for each model parameter that we wish to modify to a desired value. Our complete system of equations is (4) combined with all the constraints of type (6). However, if the number of such constraints is N_c, it is necessary (but generally not sufficient) that $N_c < N_m$ to solve the combined system of equations.

One may notice that the model constraints are of different units than (4). We therefore scale equations (6) to the same units by multiplying them by a scale factor S_m^i,

$$S_m^i \cdot \Delta m^i = S_m^i \cdot C_m^i \qquad (7)$$

For reasons to be shown, we choose this factor to be

$$S_m^i = \sqrt{\sum_i \left(A_m^{ji}\right)^2} \tag{8}$$

where A_m^{ji} is the j-th row element of A that correspond to node i and model parameter of type m.
We also define

$$D_m^i = S_m^i \cdot C_m^i \tag{9}$$

Now the second type of constraints can also be written in a matrix-vector notation

$$S\Delta m = D \tag{10}$$

For convenience, we define matrix S as a $N_m \times N_m$ diagonal matrix, with zero entries for parameters for which no constraint was defined. Vector D is defined in a similar manner.

Solution of the System

Now both matrix S and A multiply the same vector in equations (4, 10) and have the same units. Note that since $S^T = S$, the combined least squares equation reads

$$\left(A^T A + \beta S^2\right)\Delta m = \beta \cdot S \cdot D \tag{11}$$

where we also introduce an additional (positive) constant β that can control the relative weight of the model constraints vs. the traveltime constraints. Our choice of S (8) ensures us that all non-zero diagonal elements of S^2 are equal to those of $A^T A$. Thus when setting $\beta = 1$, the solution of (11) will equally respect the two types of constraints. For any other choice of β, the solution will be biased in favor of one type of constraints.
The system is over-determined and requires additional regularization terms. The final equation reads

$$\left(A^T A + \beta S^2 + C_M + L^T L\right)\Delta m = \beta \cdot S \cdot D \tag{12}$$

with C_M a damping term and $L^T L$ a structural smoothing operator. This system of linear equations is solved using the Conjugate-Gradient algorithm.

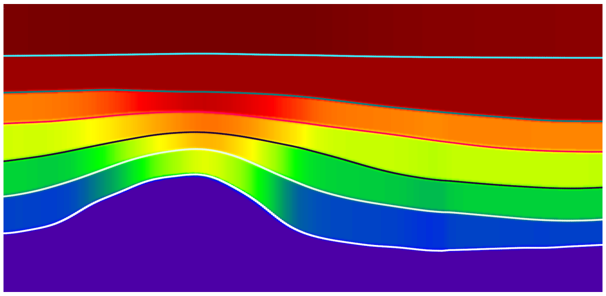
Example

As an example we used a synthetic dataset of a VTI layered model (Figure 1). It consists of 6 layers, most of which are relatively thin ($\sim$ 200m). Our initial model in this case is the correct one (i.e. the model used for generating the synthetic seismic data). The velocity and anisotropy interval parameters are constant per layer (apart from some smoothing). Perturbations were specified for two of the layers. The layer above the deepest horizon was assigned a laterally variant depth perturbation map with peak values of -180m, while the second layer from the top was assigned a constant velocity perturbation of -200m/s. Time-Preserving Tomography was applied, allowing model parameters of all types to vary (velocity, depth, ε and δ)[1]. In Figure 2 we display the velocity updated by time-preserving tomography (2(a)) compared to the initial velocity (2(b)). It is evident that the deepest horizon was pushed up and that the velocity in the second layer from the top has decreased. The velocity update in the second layer exactly matches the input perturbation (-200m/s) while the bottom horizon depth updates match the perturbations by $80\% - 100\%$. Since perturbations were specified only for two layers, all other layers received compensating updates so that traveltimes will be preserved. Also shown are seismic images obtained from Kirchhoff PSDM VTI migration using the updated velocity and anisotropy volumes (2(c)), and from migrating with the true initial model (2(d)). The updated horizon maps are displayed on top of the image and colored according to depth updates. The updated maps are the result of adding the depth updates to the initial horizon maps. They are clearly consistent with the seismic image. In Figure 3 we show migrated gathers that were created using the initial model (3(b)) and the updated model (3(a)). The gathers' lateral position is the center of the dome-shaped structure where the largest model perturbations were given (and consequently, largest updates). All events remained flat with their vertical position shifted. This indicates that the model is consistent not only with short offset data (as the stack image suggests) but also with far offset data (i.e. all rays' traveltimes are preserved). Therefore, we may call the updated model a true equivalent of the initial model.

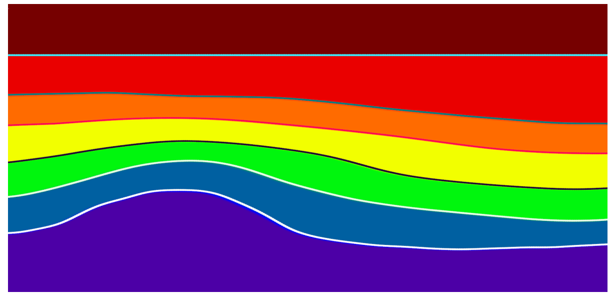
Conclusions

We present in this paper a practical method for incorporating diverse external information into the velocity model updating process, and for generating alternative anisotropic velocity models that explain the measured seismic data equally well. Our results show that the tomographic inversion is stable, accurate and remains loyal to the seismic data. Its speed of computation and flexibility make it a good candidate to be used not only in the process of velocity model updating, but also in the interpretation phase as a tool for analyzing the correctness of the current interpretation and for risk management.

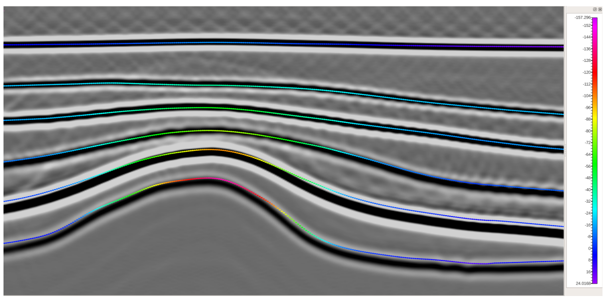
[1] More accurately, the difference $\varepsilon - \delta$ was kept fixed.



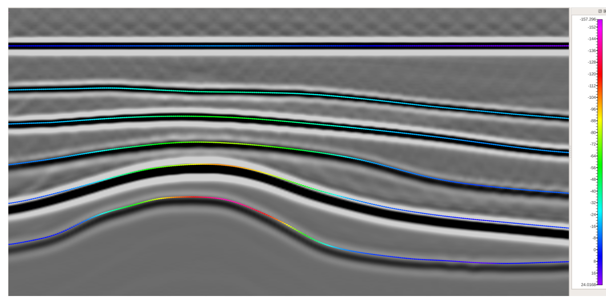
(a)



(b)

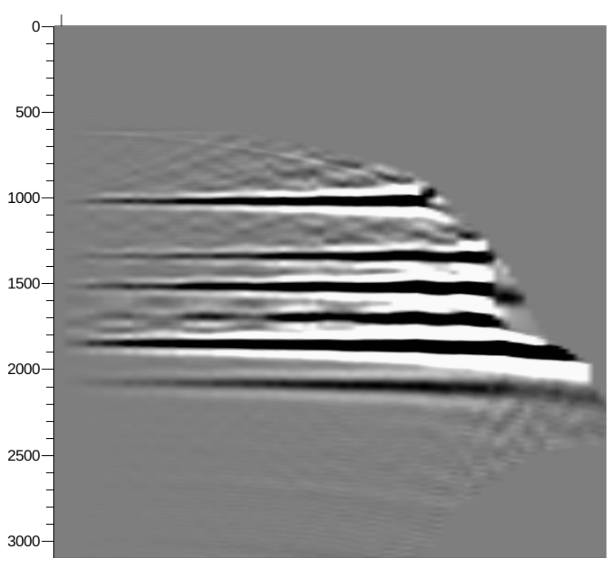


(c)

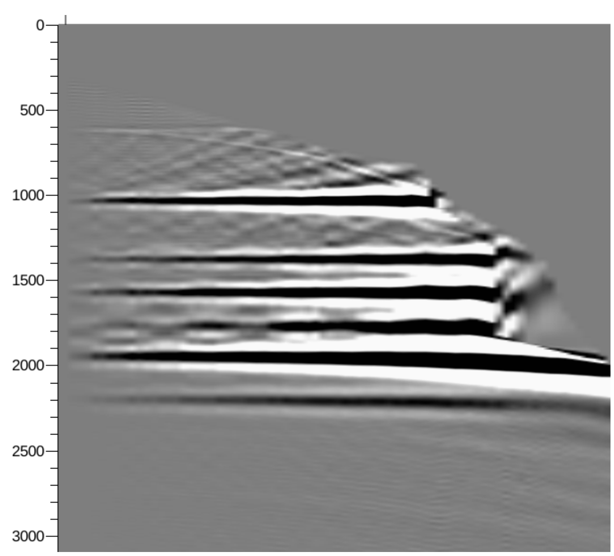


(d)

Figure 2: Comparison of the background and updated velocity model and their resultant seismic images. The background velocity model is shown in (b) on top of which are the model horizon maps. Shown in (a) is the updated model which is a result of time-preserving tomography. The updated horizon maps are also a direct result of tomography (obtained from $\{\Delta z^i\}$). A seismic image migrated using the background velocity model is shown in (d). An image migrated using the updated model is shown in (c). The model horizon maps are displayed on top of the image and colored according to depth updates. Clearly the updated model is consistent with the seismic data.



(a)



(b)

Figure 3: Depth migrated gathers using original velocity model (b) and updated velocity model from time-preserving tomography (a). The gathers correspond to the center of the dome-shaped structure (see Figure 2) where the largest model perturbations were given. All events remained flat with their vertical position shifted.

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